

## EXPERIMENT Magnetics

### Faraday's Law in Coils with Permanent Magnet, DC and AC Excitation

#### OBJECTIVE

The knowledge and understanding of the behavior of magnetic materials is of prime importance for the design of electromechanical devices such as transformers, motors, generators, and transmission lines. Hysteresis and eddy current losses reduce the efficiency of a given design. Saturation of magnetic materials makes the design procedure non-linear. This experiment verifies the concepts of Faraday's Law and demonstrates the flux concentration capacity and non-linearities of ferromagnetic materials.

#### REFERENCES

1. "Electric Machinery", Fourth Edition, Fitzgerald, Kingsley, and Umans, McGraw-Hill Book Company, 1983, Chapters 1 and 2.
2. "Applied Electromagnetics", Plonus, Martin A., McGraw-Hill Book Company, 1978.
3. "Electromagnetic and Electromechanical Machines", Matsch, Leander W., Intext Educational Publishers, 1972.

#### BACKGROUND INFORMATION

The experimental work of Michael Faraday showed that a changing magnetic field that linked a wire loop induced a voltage (emf) in the loop. The induced emf is proportional to the rate of change of the magnetic flux through the loop. The magnetic flux can change with time in many ways; the loop can be fixed in space while changing the magnetic field with time. For example, an alternating current or a permanent magnet moving back and forth through the loop can produce a time-varying field. The wire loop can also be moving or changing its shape while in a static magnetic field. The polarity of the induced voltage is given by Lenz' Law: The induced voltage causes a current in the wire loop that produces a magnetic field opposing the change in flux.

Combining Lenz' Law, which determines the sign, with Faraday's experimental results yields Faraday's Law, written in the form

$$e = - \frac{d\lambda}{dt} \tag{1.1}$$

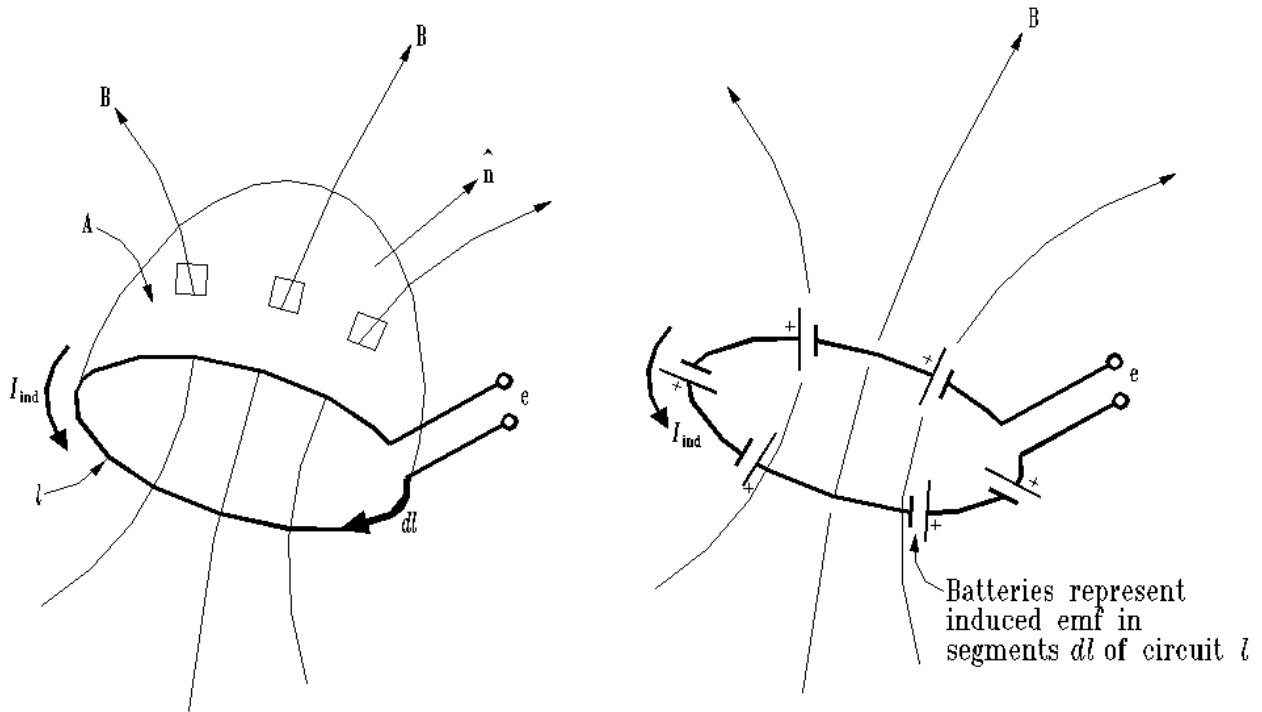
where  $e$  is the emf induced in the loop and  $\lambda$  is the effective flux linkage of the loop. The flux linkage is described by

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$$\lambda = N\phi$$

(1.2)

where  $N$  is the number of turns in the coil and  $\phi$  in the flux linking the loop.



**Figure 1:** Contour  $\ell$  coincides with a wire loop in which a voltage  $e$  is induced by the changing magnetic field  $\vec{B}$ . We can cut the wire thus introducing a small gap and bring  $e$  out to the terminals. The polarity shown is for increasing  $\vec{B}$ ; a decreasing  $\vec{B}$  will produce opposite polarity.

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If we denote the contour of the wire loop, shown in Figure 1, by  $\ell$ , the magnetic flux through such a circuit is given by

$$\phi = \iint_{A(\ell)} \bar{B} \cdot d\bar{A} \quad (1.3)$$

Thus,  $\phi$  is given by integrating the normal components of the magnetic flux density  $\bar{B}$  over any surface  $A$ , not necessarily a plane, which has contour  $\ell$  as a boundary.

Emf is related to the work done in moving a charge around a closed path. Therefore, if a voltage is induced in circuit  $\ell$  of Figure 1, a force must exist on the electric charges in order to move them around the circuit. This force must be an electric field  $\bar{E}$  which is tangential to the circuit  $\ell$  (Electric field is force per unit charge). The work per unit charge due to  $\bar{E}$ , when added around the contour  $\ell$ , must be equal to the emf induced in the circuit; that is,

$$E = \oint_{\ell} \bar{E} \cdot d\bar{\ell} \quad (1.4)$$

Combining Eqs. 1.1 through 1.4 yields the integral form of Faraday's Law:

$$\oint_{\ell} \bar{E} \cdot d\bar{\ell} = - \frac{\partial}{\partial t} \iint_{A(\ell)} \bar{B} \cdot d\bar{A} \quad (1.5)$$

Figure 1 illustrates Faraday's Law. The direction of the path  $\ell$  and the normal surface  $\bar{n}$  are related by the right-hand rule. Figure 1 shows a bowl-shaped surface  $A$ , bounded by contour  $\ell$  and threaded by an increasing magnetic  $\bar{B}$  field. Since each segment  $d\ell$  of the wire loop contributes to the induced voltage  $e$ , we can think of the induced current as being produced, at any instant of time, by a series of batteries, which are distributed along the loop, and have the polarities shown.

Figure 1 clearly shows that the terminal voltage and the induced emf have opposite polarities. Therefore, the terminal voltage is described as

$$v = \frac{d\lambda}{dt} = \frac{d}{dt} N\phi \quad (1.6)$$

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Since the devices we are working with have fixed geometries, the number of turn is constant. Thus,

$$v = N \frac{d\phi}{dt} \tag{1.7}$$

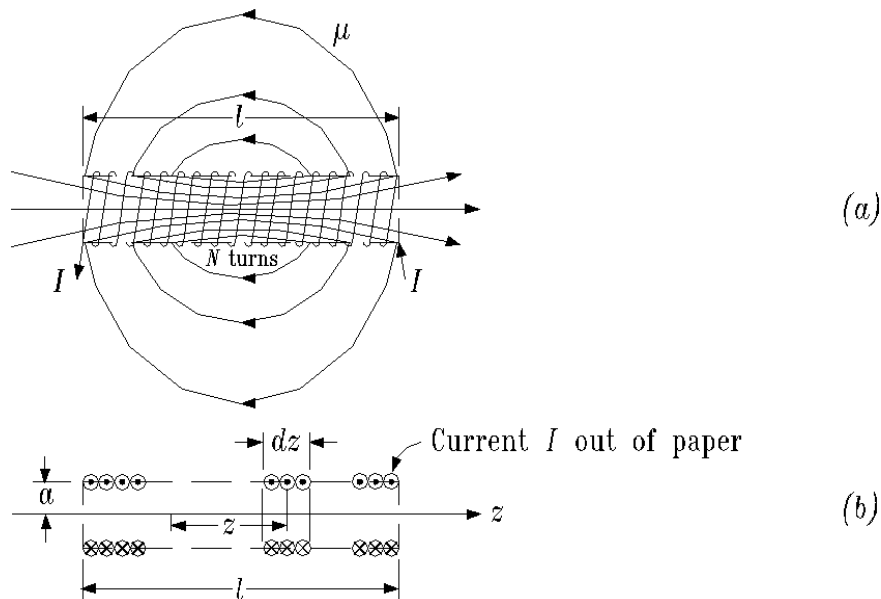
Eq. 1.7 is often called the lumped circuit form of Faraday's Law, and  $\phi$  is the average flux per turn of the coil.

To this point the coil has been used as a vehicle to support a voltage created by a time-varying flux. The coil can also be used as a magnet by applying a time-varying current to the coils. The current creates a magnetic field strength within the wire loop. According to Ampere's Law, a line integral of magnetic field taken about any given closed path must equal the current enclosed by that path; that is

$$\int H \cdot d\ell = I \text{ amperes} \tag{1.8}$$

where  $H$  is the magnetic field strength. The traditional dimension for magnetic field strength is ampere-turns/meter, therefore,

$$H = \frac{NI}{\ell} \text{ ampere-turns/meters} \tag{1.9}$$



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**Figure 2: (a) A solenoid of  $N$  turns with current  $I$  flowing; (b) a cross-section of the solenoid. The total current into or out of the page is  $NI$ , and the amount in section  $dz$  is  $NI(dz/\ell)$ .**

Magnetic field strength is related to magnetic flux density by the magnetic permeability of the medium. For an air-cored solenoid as shown in Figure 2,

$$B = \mu_o H \text{ webers/meter}^2 \quad (1.10)$$

After substituting in Eq. 1.9, the flux density becomes

$$B = \frac{\mu_o NI}{\ell} \text{ Tesla} \quad (1.11)$$

Eq. 1.11 is a valid engineering expression for coils where  $\ell \gg a$ . The more exact expression is

$$B = \frac{\mu_o NI}{(4a^2 + \ell^2)^{1/2}} \text{ Tesla} \quad (1.12)$$

For  $a \gg \ell$ ,

$$B = \frac{\mu_o NI}{2a} \text{ Tesla} \quad (1.13)$$

is sufficiently accurate. Figure 2 shows all geometries and defines all the variables.

It is obvious from Eqs. 1.9-1.13 that the magnetic field strength generated in the coil is directly proportional to the current applied. If two excited coils are placed near each other, they will be attracted toward or repulsed from each other with a force described by the following:

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$$F = \frac{B^2}{2\mu_0 d} \text{ Newtons} \quad (1.14)$$

Where  $B$  is the mutual flux density linking the two coils and  $d$  is the distance between the two coils. Obviously, the force can be either attractive or repulsive depending upon the field polarities, and the force can be controlled by varying currents applied to the coils.

In all cases of DC excitation there is no generated emf after the initial transient dies out. The transient is caused by the inductance of the coil that is defined as

$$L = \frac{\lambda}{i} \text{ Henrys} \quad (1.15)$$

Geometrically, the inductance is

$$L = \frac{N^2}{R} = \frac{N^2 \mu A}{\ell} \text{ Henrys} \quad (1.16)$$

Where  $N$  is number of turns,  $A$  is area of the flux path,  $\ell$  is length of the flux path, and  $\mu$  in the permeability of the flux path. Obviously, the inductance of a device is determined by geometry ( $N, A, \ell$ ) and material characteristics ( $\mu$ ).

From Eq. 1.15, we find that

$$\lambda = Li \text{ Weber – turns} \quad (1.17)$$

From Eq. 1.6, with Eq. 1.17, the terminal voltage required to force a current through an inductance is

$$v = \frac{d\lambda}{dt} = L \frac{di}{dt} + i \frac{dL}{dt} \quad (1.18)$$

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if the coil is ideal or of a super-conducting material. Real coils, such as those in the laboratory, have resistance that also must be overcome when forcing current through the coil.

Therefore,

$$v = Ri + L \frac{di}{dt} + i \frac{dL}{dt} \quad (1.19)$$

where  $R$  is the coil resistance.

The laboratory coils have a fixed number of turns that causes the third term on the right side of Eq. 1.19 to go to zero. In addition, basic circuit theory tells us that the second term on the right side of Eq. 1.19 goes to zero after the transient, 5-6 time constants. Therefore, for all practical purposes the current through the coil with DC excitation is

$$I = \frac{V}{R} \text{ amps} \quad (1.20)$$

regardless of the permeability of the magnetic path.

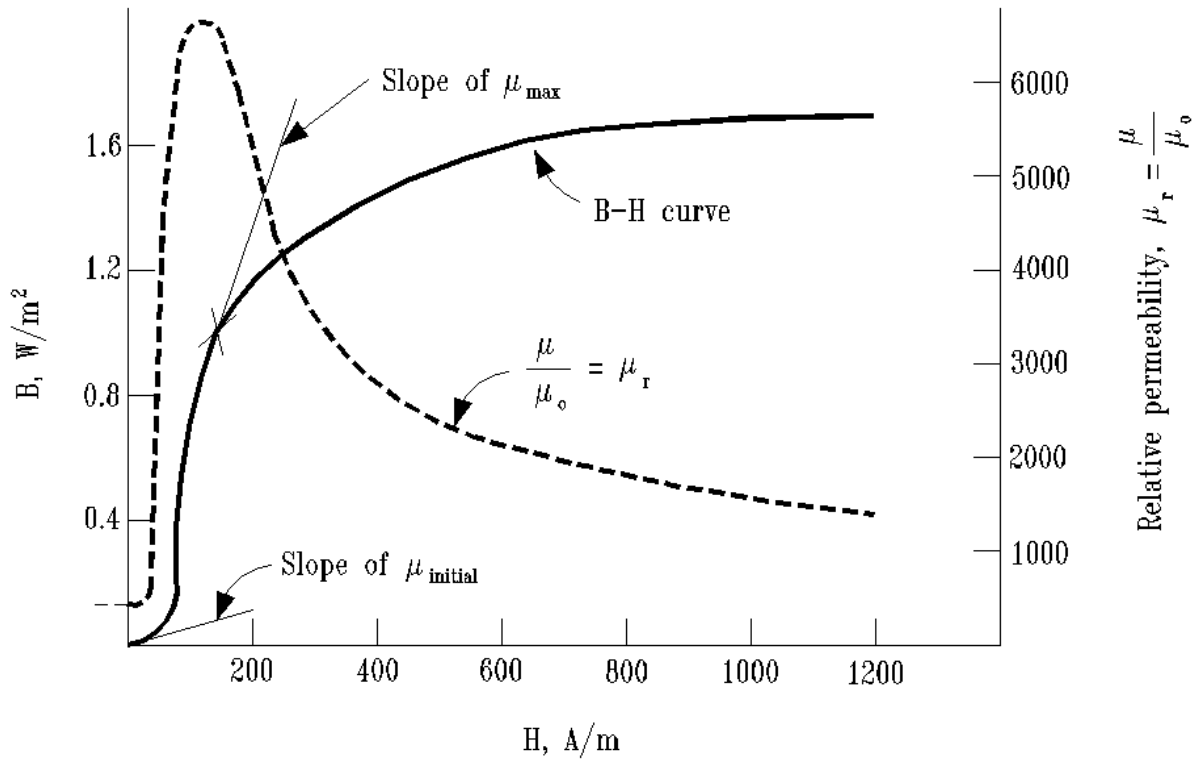
The permeability of the magnetic path is determined by the type of material in the path. If the medium is air, or some other non-magnetic material, such as wood or glass, the permeability is

$$\mu = \mu_o = 4\pi \times 10^{-7} \text{ Henrys/meter} \quad (1.21)$$

Permeability relates the flux density in the magnetic path to the magnetic field strength required to achieve that density, thus

$$\mu = \frac{B}{H} \text{ Henrys/meter} \quad (1.22)$$

Obviously, if the medium is non-magnetic, the ratio described by Eq. 1.22 is constant.



**Figure 3: Magnetization curve of commercial iron. Permeability is given by the ratio  $B/H$ .**

If a ferromagnetic material, such as iron, is placed in the flux path, we find that much more flux is produced for a given current. Figure 3 shows a typical magnetization curve for commercial iron. On the curve

$$\frac{B}{H} = \mu = \mu_r \mu_o \text{ Henrys/meter} \tag{1.23}$$

where  $\mu_r$  is the relative permeability of the material. Figure 1.3 shows a peak relative permeability exceeding 6000, which means for a given current, the iron core will have 6000 times the flux density of an air core. A referral to Eq. 1.16 shows that the inductance of the coil will be 6000 times greater if the applied current causes the system to operate at the point of maximum relative permeability. For DC excitation, where the current is constant, the inductance is controlled by varying the cross sectional area of the iron which causes the flux density to vary.

When AC excitation is applied to the coil, the results are different. The third term on the right side of Eq. 1.19 can still be set to zero, but the second term on the right side cannot be zero



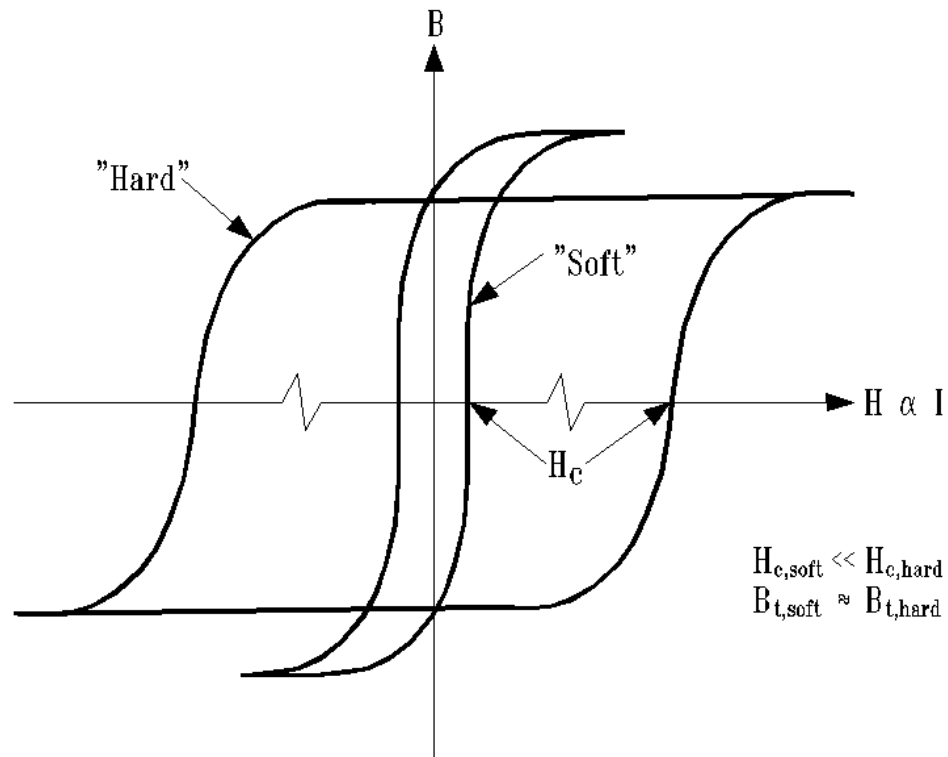
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because the current is now a time-varying quantity. The second difference from the DC case is the addition of a time-varying flux and a non-zero terminal voltage, as described by Eq. 1.7, appearing across the coil. If we assume that the coil has an air core and that, for the time being, the resistance is negligible, then the total applied voltage appears across the coil and causes flux to be produced. The flux must be such that its time derivative multiplied by the number of linked turns exactly matches the applied voltage. If we further assume that the applied voltage is sinusoidal, then the flux must vary as a cosine such that

$$\frac{1}{N} \int v(t) dt = \phi(t) \text{ Webers} \tag{1.24}$$

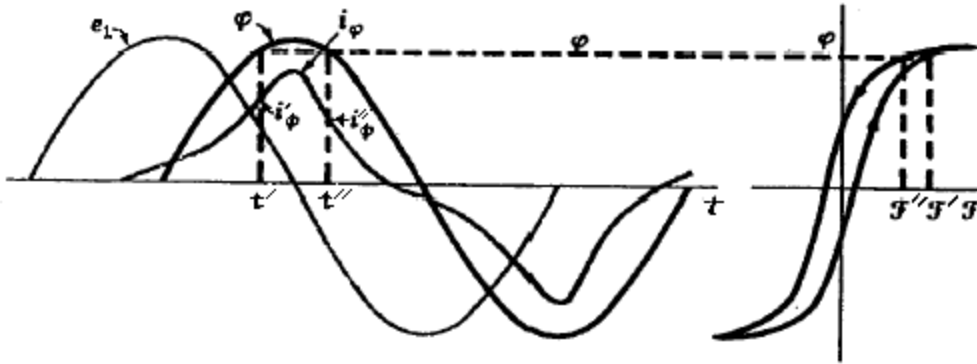
Magnetic flux is actually produced by current (Eq. 1.8), so Eq. 1.24 must imply that the power supply which is producing the sinusoidal voltage must also supply a co-sinusoidal current. This current must have sufficient magnitude to produce the required flux.

The flux and current are related by the permeability of the magnetic path, which in turn is a function of the material used for the magnetic path. If the path consists of ferromagnetic material, it takes much less current to produce the required flux. In fact, if the voltage applied to the coil is held at a constant RMS value, the required flux also has a constant RMS magnitude. The current required to produce this constant flux varies inversely with the relative permeability of the magnetic path.

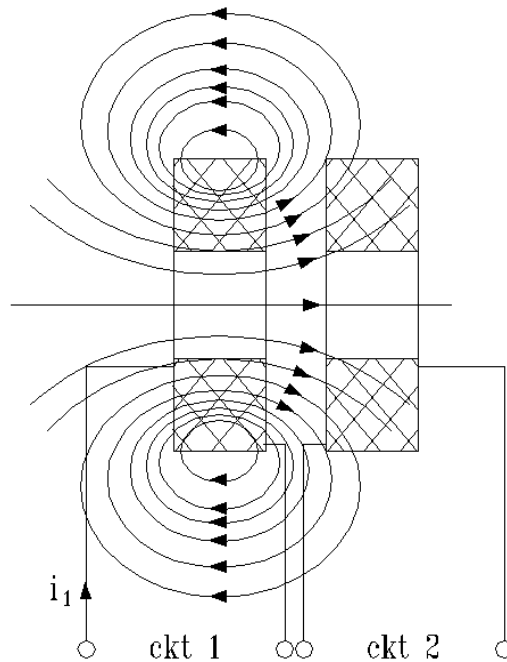


**Figure 4:** Hysteresis loops of soft magnetic materials, which are easy to magnetize and demagnetize, and those of hard magnetic materials. The former is useful in transformers and machinery, whereas the later finds application in permanent magnets.

The flux and current relationship is made more complex by the non-linear characteristics of ferromagnetic materials, as shown in Figure 3. In addition, ferromagnetic materials exhibit hysteresis characteristics shown in Figure 4. Since, in most cases of AC excitation and particularly in a teaching laboratory, the strength of the AC source far exceeds that of the load, the source voltage tends to maintain its sinusoidal wave shape. As discussed above, the flux must maintain its sinusoidal nature. There, any non-linearities due to the flux path will appear in the current wave shape. Figure 5 shows the result of a contained flux wave on a non-linear iron flux path. The current shows a definite third harmonic flux wave on a non-linear iron flux path. The current shows a definite third harmonic component. Note that the third harmonic is caused by a stiff sinusoidal voltage source which causes an essentially sinusoidal flux within a non-linear magnetic path.



**Figure 5:** (1) Upper half of energy loop. (b) Sinusoidal flux and exciting current waves. (c) Waves of induced voltage, flux, and exciting current.



**Figure 6:** Two circuits with mutual inductance.

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Figure 6 shows two coils sharing a common flux; called mutual coupling. If circuit 1 of the figure is excited by an AC source, then the flux linking circuit 2 is time varying. The flux linking circuit 2 causes a voltage to appear across the terminals of circuit 2 (Faraday's Law). Thus, we are seeing an indication of magnetic flux in circuit 2 due to the excitation current in circuit 1. This is the concept of mutual inductance, which, in this case, is the inductance from circuit 1 to circuit 2, thus,

$$L_{12} = \frac{\lambda_2}{i_1} = N_1 N_2 \frac{\mu A}{\ell} \text{ Henry} \quad (1.25)$$

where  $N_1$  and  $N_2$  are the effective number of linked turns on each coil,  $A$  is the area of the magnetic path, and  $\ell$  is the length of the magnetic path. It is seen from Figure 6 that the effective number of linked turns can be changed by varying the mutual geometry of the coils. Eq. 1.25 notes the effects of coil geometry and permeability.

### SUGGESTED PROCEDURE

1. Connect experimental set-up shown in Figure 7. Low voltage coils are found in the laboratory shelf for each bench; they're the coils with visible windings (use the large wire coil). The permanent magnets and integrators are in the drawer on the lower side of the bench. Set the oscilloscope using the procedure below. Then move the magnet in and out of the coil at two speeds and different polarities. Record  $|v|$ , and  $|\phi|$  amplitude Record waveform  $v$ , and  $\Phi$

Steps for configuring the oscilloscope

Select Default Setup

Select CH\_1 Menu

Coupling = DC, BW Limit = Off, Volts/Dev = Coarse

Probe: > Voltage > Attenuation = 1X > Back

Invert = Off

Adjust the Volts/Div knob to 100 mV / Div

Select CH\_2 Menu

Coupling = DC, BW Limit = Off, Volts/Dev = Coarse

Probe: > Voltage > Attenuation = 1X > Back

Invert = Off

Adjust the **Volts/Div** knob to **50 mV / Div** Set the **Horizontal**

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**Sec/Div = 100mS** Select **Trig Menu**

**Type=Edge, Source=CH\_1, Slope=Rising, Mode=Normal, Coupling=DC**

Adjust Trigger Level knob: **Level=** between **20mv** and **100mv**

Select **Single Seq**

Press **Ch\_1** in the vertical menu buttons

Press **Measure**, and **Select Measurement** for Ch\_ 1 top button. Press **type** until, **Pk-Pk**, then **Back**.

Press **Ch\_2** in the vertical menu buttons and select **Pk-Pk** again Press **Measure**, and **Select Measurement** for Ch\_ 2 second button. Press **type** until, **Pk-Pk**, then **Back**.

Record the voltage and flux waveforms and record  $|v|$  and  $|\phi|$ .

2. Using the coil with the largest wire, connect the circuit shown in Figure 8. There are two DC power supplies on the console. They are all the same, so use the most convenient. To set the DC power supply follow the following procedure:

Be sure all knobs are turned fully counter-clockwise for voltage and at the 9:00 position for current. Turn on DC supply. Increase voltage level slightly until DC ammeter gives a read-out. Adjust current knob until 3.5 Amps (safety current limit) register on the Ammeter set the Ammeter to **DC current**. Decrease voltage level to zero and remove short-circuit across the coil.

Increase the voltage level. Using the voltage control set current to 3 amperes, energize the coil and observe the force on the permanent magnet. Reduce current to 2 and 1 Amps magnitudes and different magnet ends. Use the right hand rule to fine the magnetic polarities of the field in the coil. Observe the forces of attraction and repulsion so that you can determine the magnet's polarities. As you run these experiments, keep the general concepts of electromagnetic forces and energy conversion in mind.

3. The circuit show in Figure 9 is used for experiments required AC excitation. All equipment taken from the shelves must be replaced on the shelves at the completion of the laboratory period. The voltage across the  $1\Omega$  (found on the equipment shelf) is directly proportional to the current through the resistor.

Now place the two coils directly next to each other in the position of maximum coupling. This position creates maximum coupling because the flux that passes through any given surface area of one coil passes through any cross-sectional area of the other coil. Adjust

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the Single-Phase AC Source so the **RMS current is 3 amps**, make sure the Ammeter is set to **AC current**. **Note the RMS voltage (less than 6 Vrms)** across the excited coil (primary Heavy wire) because during the next procedures this voltage should be kept constant. Insert three iron strips at a time, through the windows of both coils and record RMS values for voltage, current, and flux while **maintaining constant voltage**. Also, record the voltage and flux waveforms as the number of iron strips is varied.

No. strips	Voltage RMS	Current RMS	Flux
0			
3			
6			
9			

Switch the oscilloscope to X-Y mode. To do this press the **DISPLAY** button and select Format from the horizontal menu and XY from the vertical menu. The  $\phi$  versus  $I$  curve is displayed on the screen. Record the waveform of the  $\phi$  versus  $I$  curve for air and various numbers of steel strips while **keeping the voltage constant**.

If the hysteresis curve contains some unexpected loops, adjust the 100 $\Omega$  potentiometer, this should permit filtering them out. Record the waveforms and observe the changes in the hysteresis loop using 3 iron strips when varying the **terminal voltage** for three different values.

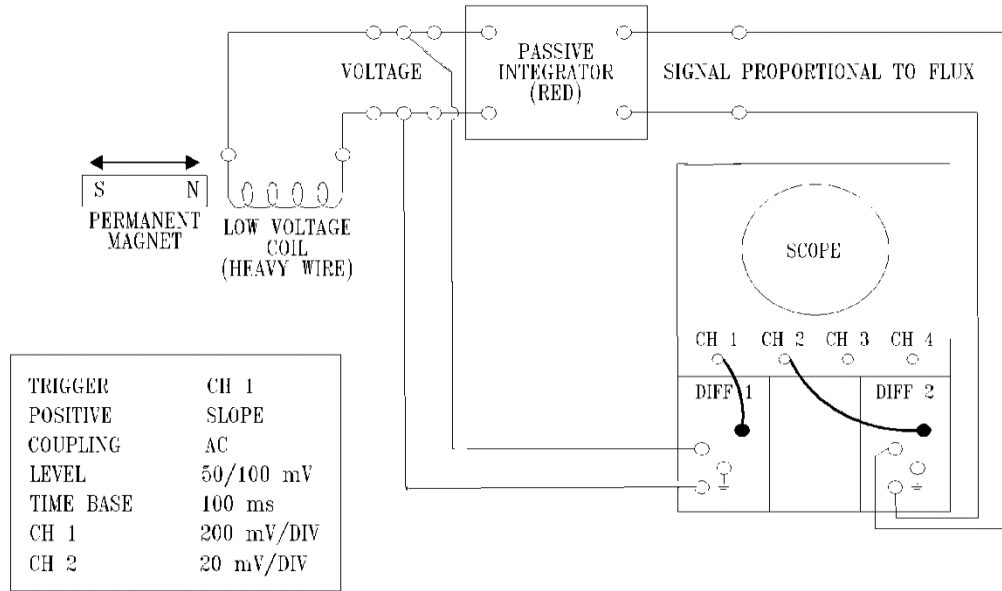
Record the hysteresis waveforms when **biasing** the 3 iron strips at one end with a **permanent magnet**.

Using only 3 iron strips verify that the **permeability** of saturated iron is the same as the permeability of air (observe the 3 amp current limit).

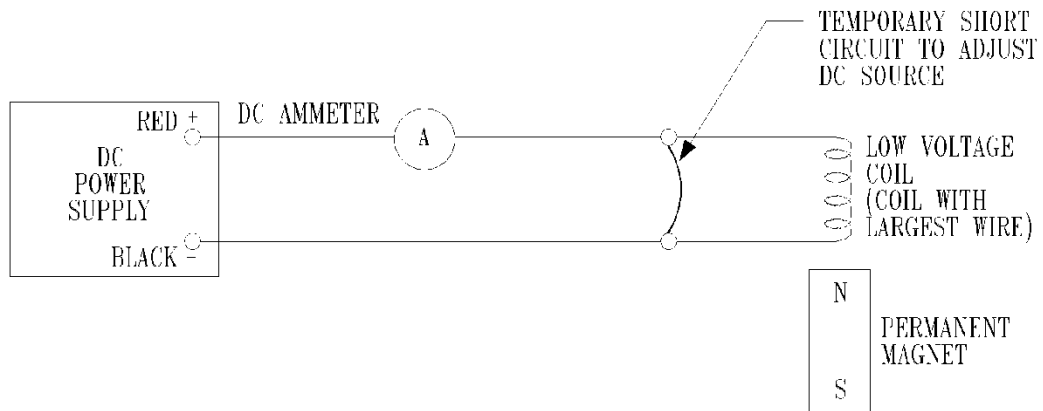
**REPORT follow the template**

1. Explain the observations made in Part 1 in terms of Faraday's Law. Note particularly the factors affecting the polarity, magnitude, and shape of the voltage and the relationship between voltage and flux.
2. Explain the results of Part 2 in terms of magnetic polarities, current magnitudes, and the general concepts of electromagnetic forces.
3. Explain the observations of Part 3 in terms of mutual coupling and permeability. Why is the terminal voltage across the coil held constant? What happened to the current as iron was added or subtracted? What happened to the fluxes? Describe the final observation in terms of Figure 3. If you are designing an electromechanical device that requires linear operation, what parameters need to be considered? How can you modify your design to minimize eddy currents, and in return, how does this modification affect your device?
4. Using your knowledge of electromechanical devices and the results of this experiment, what are some physical entities that affect saturation? Also, what design parameters can be altered to prevent saturation?

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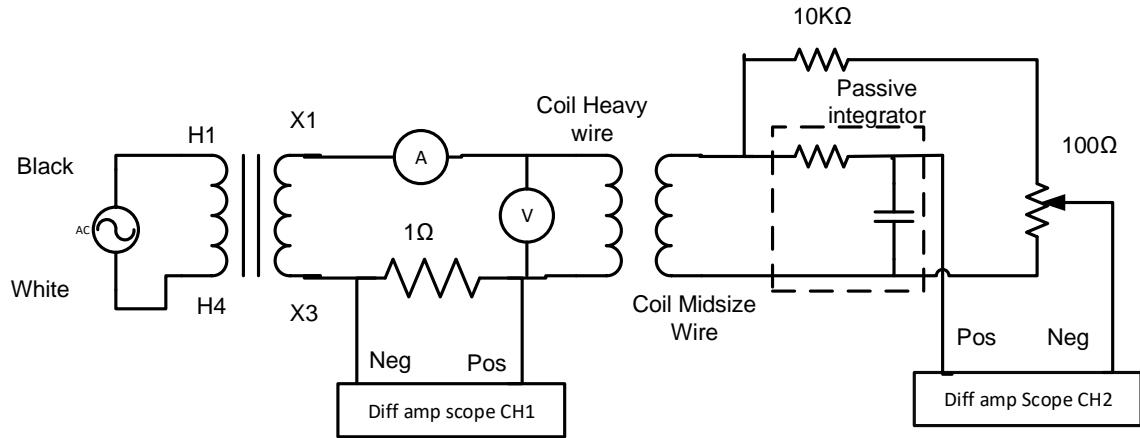
**Figure 7**



**Figure 8**



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**Figure 9**